Commodity Bonds: A Risk Management Instrument for Developing Countries

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I. INTRODUCTION *

In the 1980-86 period commodity earnings fluctuations increased and commodity prices declined sharply (in real and in nominal terms), adding to the difficulties of managing the balance sheets of firms in the commodity business. The purpose of this paper is to bring together and review disperse information on a financial instrument that could prove useful in managing the balance sheets of commodity-producing or commodity-using firms: the commodity bond. This instrument is of particular interest to companies that have low operating costs but high debt servicing requirements. 1/ Commodity bonds can also be useful in restructuring debt. By linking commodity earnings with interest expenditures, commodity bonds may reduce the deleterious impact of lower commodity prices and higher interest rates on net worth. 2/ With commodity bonds opportunity gains due to higher commodity prices (above a defined level) can be foregone in exchange for lower interest rates. For the borrower, commodity bonds issued at par represent an instrument with lower coupons than similar corporate straight bonds. For the investor, lower

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1/ Commodity bonds can be issued by public (governmental) as well as private entities.

2/ Liquidity problems arise when net worth declines (due to lower commodity prices or higher interest rates) indebtedness increases and/or real assets decline.
interest receipts during the life of the bond are often compensated by a premium (in addition to the principle) at maturity; they also represent insurance for satisfying future consumption of the commodity in question.

Over many years the developing countries highly dependent on exports of primary commodities have been seeking ways of obtaining more stable export prices. For several commodities (oil and bauxite) cartels were formed, while for some others international commodity agreements were negotiated. These efforts have seldom resulted in long-term stability of prices. At the same time, there has been a proliferation of financial instruments which corporations and government bodies--mostly in the industrial countries--have been using to hedge their commodity risk and fund-raising efforts. One of these instruments is the commodity bond. The question arises: can these financial instruments be used in a similar role by developing countries?

The following sections review the literature and provide answers to the following questions: What is a commodity bond? What is the nature of the demand for commodity bonds? What is the optimal hedging amount? How are commodity bonds priced? Finally, there is a review of the financing needs of the developing countries and of the contributions to these needs which various financial instruments--including commodity bonds--could make. 1/

II. WHAT IS A COMMODITY BOND?

A commodity bond is a financial security in which the return (yield to maturity) is linked *inter alia* to the price of its underlying commodity (see O'Hara, 1984). Conventional bonds pay a stated nominal interest rate (coupon) and a stated nominal amount upon maturity (principal). The commodity bond payoff is a stated quantity of a particular commodity. (For example, a US$1,000 face value gold bond may be redeemable at maturity for 2.50 troy ounces of gold). The interim interest payments may or may not be likewise denominated in units of the particular commodity. Some commodity bonds also incorporate an option feature by allowing the holder to receive either the nominal face value or the designated commodity amount at maturity. These commodity bonds are often called commodity convertible or indexed bonds. Some recent commodity bond issues allow the holder to receive the nominal face value and to choose whether to exercise an option to buy (or sell) a certain amount of the designated commodity at a predetermined price (exercise price) at maturity (or at any other predetermined dates during the life of the bond). These commodity bonds are called commodity-linked bonds.

It is the quantity-denominated return structure that distinguishes commodity bonds from conventional bonds. With a conventional bond the nominal return is known but the real return is not. In respect of the commodity bond, both nominal and real monetary returns are unknown. The uncertain real return of a commodity bond also differentiates it from an index bond; that is, a bond whose return depends on an aggregate price level index. Index bonds protect the holder from changes in the overall price level while commodity bonds protect the holder from changes in relative price levels. Only if prices in
the price index change by exactly the same amount as that of the commodity price will these financial instruments be identical.

Commodity bonds differ from forward contracts. One difference lies in their cash flows. The purchasers of a commodity bond pay the seller at the outset, while in a forward contract no such exchange occurs until the completion of the contract. Further, commodity bonds typically pay interim coupon payments, but forward contracts pay off only at maturity. Forward contracts are primarily short-term agreements. 1/ Commodity bonds are designated as long-term instruments. In a one-period world, a short position in a commodity bond, i.e., issuing a commodity bond, is equivalent to a short position in a forward contract for the commodity plus a short position in a money bond. Similarly, a long (purchaser) position in a commodity bond is equivalent to a portfolio consisting of a long bond and a long forward contract.

In a multi-period setting with perfect capital markets, this same relationship holds if both the commodity bond and the money bond are pure discount securities. 2/ Replicating a coupon-bearing commodity bond, however, is more complicated. A long position in a commodity bond is equivalent to a long position in a money bond and a long position in a portfolio of forward contracts selected to match the cash flow of the money bond. These portfolios are dependent on using long-term forward contracts to replicate the commodity bond's quantity payoff. Such long-term contracts are, however, not commonly

1/ Long-term interest and exchange rate forward contracts are being used increasingly in swap arrangements.

available as investment vehicles. 1/ In the forward contract both the long
and the short sides have unfulfilled obligations. The short promises to
deliver a certain quantity of the commodity; the long promises to pay a fixed
price for it. By contrast, in a commodity bond the long meets all of its
obligations when the bond is issued; only the issuer has an unfulfilled
obligation.

Similarities and differences exist also between commodity bonds and
future contracts. Both securities involve making or taking delivery of a
specific commodity at a specified date in the future. Unlike futures,
commodity bonds do not trade in organized exchanges. Margin requirements
are also an important feature/consideration in futures contracts in contrast to
forward or commodity bond contracts. In addition, the cash flow timing is
significantly different. Most futures contracts are reversed prior to
maturity, and hence, the commodity is seldom actually delivered. In a forward
contract and in a commodity bond, however, delivery is more likely to occur.
Lastly, futures contracts are also short-term contracts. Therefore, in the
absence of long-term forward and futures contracts, commodity, convertible or
indexed bonds are unique financial securities. 2/

1/ Long-term forward contracts do exist in some industries. For example, a
nuclear power plant might arrange for delivery of uranium via such a
contract. The integrity of each side is crucial and only the most
established companies are likely to be involved. See Joskow (1977) for
some problems thereon.

2/ A proliferation of commodity bonds could add liquidity in commodity
markets. If the straight bond part is separated from the commodity part,
the longer-term forward markets could be developed as has been done in the
case of interest rate and exchange rate swaps.
A commodity bond could appear also in the financial markets as a straight (or zero) bond with a commodity option/warrant attached to it at the payoff of the principal. The ("call") option gives the right to the bondholder to buy a pre-agreed quantity of the commodity at a pre-agreed price (exercise price). \(^1\) The option is bound to be exercised if the actual price at payoff is higher than the exercise price. The bondholder could then make a profit by exercising the option and selling the commodity in the spot market. This premium increases the yield to maturity of the bond. Caps and floors also can be established in a commodity bond to constrain the variability of the implied yield to maturity. The market for long-term commodity options could be greatly developed with the proliferation of these instruments. Following are three recent examples of commodity bond issues.

During 1980, Sunshine Mining Company, operator of the largest silver mine in the United States, made two US$25 million bond issues backed by silver. Each US$1,000 bond is linked to 50 ounces of silver, pays a coupon rate of 8½% and has a maturity of 15 years. At maturity the company promises to pay the bondholders either the US$1,000 face value or the market value of the 50 ounces of silver, whichever is greater. At the time of the first issue in April 1980, silver was trading at US$16 an ounce so that the value of 50 ounces was US$800.

In August of 1979, an agency of the Mexican Government issued bonds in local currency backed by oil. Each 1,000 pesos bond was linked to 1.95354 barrels of oil, had a coupon rate of 12.65823% and a maturity of three years. At maturity they were to be redeemed at face value, plus the amount by

\(^1\) "Put" options also may be attached to the bond.
which the market value of the reference oil bundle exceeded the face value and plus all coupons received during the life of the bond if this amount were positive. This was the third successful issue of "petrobonds" by the Mexican agency.

In August 1985, a minute (US$6.6 million) convertible eight-year fixed rate non-callable bond issue was brought to the market by a six-member lead management team. The beneficiary of the issue was the Pegasus Gold Corporation, a U.S. gold and silver mining concern. A subsidiary of Bank Gutzwiller Kurz, called Peggold Overseas Limited, was incorporated in the Cayman Islands with a share capital of US$1,000. Peggold issued the bonds. Its obligation to repay the principal was secured by a pledge of a zero coupon bond issued by General Electric, maturing at the same time as the Peggold notes. The five percent coupon on the notes was secured by a joint and several guarantee of two members of the Pegasus Gold group. As a guarantee fee, the two guarantors were paid the net proceeds of the issue after deduction of the zero coupon bond's purchase price. The innovations in the Peggold issue were in the denomination of the individual bonds. The issue was gold-backed. This did not mean that the issue was secured by a pledge or deposit of any physical gold; instead the investors were given two special options, exercisable at any time during the life of the issue.

The first option enables an investor to take his bond along to any one of a number of exchange agents in Europe and ask for delivery by Pegasus of physical gold bullion. The amount of gold bullion per bond which he is entitled to is 100 grams or the then market price of 100 grams of fine gold bullion. The second option allows the investor at any time to obtain the
redemption of his bond for cash. Instead of being entitled to US$1,065 (the principal), he would receive the cash value of 100 grams of fine gold bullion.

The effect of these two put options is to turn the Pegasus issue into a hedge against movements in the market price of gold. If the gold price rises over time, an investor can exchange his bond for gold, sell the gold and make a capital gain over his original investment of US$1,065. Alternatively, he could cut out the step of receiving and selling the gold and simply receive his investment back plus his capital gain. If the gold price falls over time, the investor has the right to keep the bond, receive the coupon payment and await the redemption date to be paid at par.

The first two commodity bonds are examples of commodity convertible bonds while the last is an example of a commodity-linked bond. In the 12 months ending October 1987 there were some 45 commodity bond issues. Some of these issues were public and some were private. Almost ninety percent of these issues were linked to gold or silver; ten percent were linked to oil while the remainder was linked to copper and zinc. Ninety-five percent of these issues were of the commodity-linked type while the remainder were of the commodity-convertible type. None of the issues involved a developing country public or private organization. Two of the issues involved a public entity while all others involved private companies.
III. THE DEMAND FOR COMMODITY BONDS

This section reviews the literature on the nature of the demand for commodity bonds. In particular, the analysis focuses on the work of O'Hara (1984) on consumption risks with commodity bonds. O'Hara asserts that as commodity bonds can hedge against changes in relative prices, they provide a mechanism to hedge risks attached to future consumption. He analyses the desirability and limitations of this hedging ability in the context of a dynamic programming model. In particular, he shows that while commodity bonds protect against relative price changes, they do so by introducing variability in the future real income stream. If commodity bonds are priced fairly, they will be demanded only if there is a minimum necessary consumption quantity or the bond's payoff negatively correlates with the individual's portfolio return. In either case the bond is valuable because it provides a form of insurance in hedging risks of future consumption. This insurance aspect of the commodity bond provides some interesting insights into both the economic role of commodity bonds and individuals' hedging behavior. As with any insurance contract, a commodity bond is only valuable if the protection it provides outweighs its cost (premium). In the case of hedging consumption risks introduced by changes in relative prices, this protection will be valuable when the consumer must purchase a particular good and may not always have sufficient income to do so. If the consumer can substitute other goods, however, he has no need for commodity-specific insurance. Perhaps more important, if the consumer's income is high enough, he would be better off self insuring (or even, if possible, providing such insurance to others). This suggests that to protect future consumption, what is needed is not "price insurance" but rather "income insurance".
O'Hara derives the demand for commodity bonds through a utility maximization problem. The derived demand function indicates that a risk neutral consumer will hold only enough commodity bonds to guarantee survival. A risk averse consumer will demand commodity bonds above the minimum level. The less risk averse the individual, the fewer the bonds he will hold. The other characteristic of the demand for commodity bonds is that as income increases, the consumer decreases his demand for commodity bonds. If his income is high enough, even the most risk averse individual will demand zero commodity bonds. Indeed, as it is also noted in the previous paragraph, in the absence of short sale restrictions, a wealthy risk averse individual would even issue a commodity bond. The demand for commodity bonds function also indicates that there is a maximum of commodity bonds that a risk averse consumer would want, no matter how risk averse he is. If there was no minimum consumption, there would be no demand for the commodity bond. Provided the demand curve is downward sloping, a utility maximizing individual's demand is already partially hedged with respect to price changes. An increase in the price of the good results in the demand for goods falling. Hence, the quantity demanded will change with shifts in relative prices. If an absolute minimum consumption quantity is always demanded, this hedging property will be absent and the hedging ability of the commodity bond may become valuable. The commodity bond can protect the consumer from being unable to purchase the needed consumption quantity. The commodity bond therefore provides a form of "price insurance". If there is no minimum consumption quantity (or if we consider an individual whose income is always sufficient to purchase such a minimum quantity), however, the insurance provided by the commodity bond would come at a very high price. The reason is that a commodity bond affects a
consumer's future period income. While the commodity bond provides the same quantity of the good, independently of the price level, the real value of the quantity to the consumer depends on both the commodity's price and the consumer's overall price index. 1/ As relative prices change, the optimal quantity the consumer demands also changes and this in turn affects the consumer's price index. The index change distorts the real payoff of the commodity bond and the loss incurred by holding commodity bonds when prices are low is greater than the gain from such a position when prices are high. 2/

The gains to real income when prices are high are more than offset by the losses to real income when prices are low. 3/ The "price insurance" provided by the commodity bond thus comes at the expense of greater real wealth variability. This wealth variability problem can be mitigated, however, if the commodity bond's price reflects the real purchasing power effect the bond has on the consumer. The bond would have to sell at a price that compensated the consumer for the increased risks it imposed. Based on a Stone-Geary utility function, O'Hara estimates this insurance premium to be

\[
\frac{\text{cov}(P, p^{-a})}{(1 + r) E(p^{-a})}
\]

1/ i.e. the price of the "basket" of goods.
2/ i.e. with respect to the commodity prices at the time of the purchase of the commodity bond.
3/ For a mathematical interpretation of these results, see O'Hara (1984) and Richard-Sundaresan (1981).
where $P$ is the price of the commodity in the payoff period; \(^1\) where the premium is described by a simple discrete distribution in which $P$ equals $k$ with probability $q$ and $m$ with probability $(1-q)$; where $a$ is a parameter of the utility function (it is the elasticity of the utility with respect to the commodity consumed); and where $1 + r$ is the return of a conventional bond (yield to maturity).

The most important result of the O'Hara study, however, is that price variability is not the overriding factor in the decision to hold commodity bonds. It is the overall wealth variability and its correlation with the bond's return that influences demand. If wealth variability is large (i.e. income fluctuates greatly between periods), the demand for commodity bonds will increase if the correlation between wealth and bonds is negative (low income, high return, etc.); and the demand for commodity bonds will decrease if the correlation is positive (low income, low return, etc.). Hence, commodity bonds are desirable to the extent that they offset changes in stochastic income. Since changes in real wealth affect consumption, commodity bonds can hedge risks to future consumption by "smoothing out" this income stream. The demand for commodity bonds is thus similar to the demand for any other investment vehicle; its value depends upon the role it plays in a diversified portfolio.

\(^1\) In the O'Hara paper only two periods are considered.
IV. THE OPTIMAL HEDGE

In Section II it was shown that there are two parts of a commodity bond: the corporate-straight bond part and the commodity option or the forward contract part (in the commodity-linked and the commodity convertible bonds respectively). In the contract of their risk management strategy, the issuers of the bonds have to optimize not only their financing needs and their interest rate risk management needs (as reflected by their net supply of corporate bonds), but also their commodity risk management needs (as reflected by their net supply of commodity options/forward contracts).

In this section the economic literature in the area of the determination of the optimal hedging requirements of commodity producers is reviewed. 1/ Commonly, hedging is defined as the temporary purchase or sale of futures/options contracts (or forward contracts if available) in order to offset the price risks arising from commitments in the cash market. Because more hedgers are short than are long, the traditional view was that short hedgers paid a risk premium to long hedgers and long speculators which was reflected in a downward bias of the futures price called "normal backwardation". Working (1953) rejected this relatively passive view of hedging, as he observed that most hedging is discretionary, i.e., it is initiated with the intention of profiting from a favorable movement in the spot/futures basis. Johnson (1960) and Stein (1961) developed portfolio theories of hedging within which Working's theory is a polar case. Whether short hedgers pay a risk premium remains controversial (see Goss and Yamey

(1978) and Sharpe (1981) for reviews and Anderson and Danthine (1983) for a theoretical analysis. The implication from Dusak (1973) using the capital asset pricing model (CAPM) is that commodity risks are diversifiable and hence require no premium, but Breeden (1980) has estimated significantly non-zero betas from commodities in a more general version of the CAPM.

The theory of hedging as applied to producers was developed by McKinnon (1967) and extended by Peck (1975), Heifner (1978), Rolfo (1980), Anderson and Danthine (1983), and Gemmill (1985) among others. A country or a firm begins with a given portfolio of expected exports which have both price and quantity risks. Futures/options are then bought or sold in order to adjust the expected return/risk characteristics of the export portfolio which is considered quite separately from other parts of the country's (firm's) asset portfolio. Rolfo (1980) and Gemmill (1985) show that the optimal hedge is not the entire expected output. How much it is optimal to cover depends on the solution of a utility maximization problem. 1/ The existence of different quantity risks and different product mixes supports the view that the contribution to portfolio risk of a given product may differ widely across countries and firms. 2/

1/ For application and examples, see Rolfo (1980) and Gemmill (1985).

2/ Imposing a single level of price stabilization on all exporters of a product through an International Commodity Agreement is therefore likely to be highly inefficient relative to an adjustment via futures trading because the latter allows for individual choice according to the degree of risk aversion. Nevertheless, the Gemmill (1985) results suggest that the alternative of hedging with one year futures has only moderate potential for stabilizing export revenues. Futures or forward contracts of longer than one year are needed.
Benninga, Eldor, and Zilcha (1985) extend the theory of optimal hedging by deriving production and optimal hedging rules for an exporting firm which faces both commodity-price and foreign-exchange-rate uncertainty. He shows that the size of the commodity hedge is independent of the properties of the foreign-exchange market. However, the optimal foreign-exchange hedge depends on the commodity hedge and the properties of the commodity forward markets. The firm's (country's) production decision is independent of its objective function if both forward markets exist but depends on the consumption beta of the unhedgeable risks in the absence of one or both of the markets.
V. PRICING OF COMMODITY BONDS

The model for pricing commodity-linked bonds uses the option pricing framework as pioneered by Black and Scholes (1973), extended by Merton (1973) and Cox and Ross (1976) and further refined by Schwartz (1982). The model for pricing commodity-convertible bonds uses the option pricing framework of commodity-linked bonds \(^1\) or the model of pricing convertible bonds as presented among others by Brennan and Schwartz (1980). As commodity-convertible bonds are equivalent to appropriately specified commodity-linked bonds, discussion here focuses only on the latter type of bond. The key assumption of the model is that the underlying commodities, the commodity-linked bonds and the equities of the firm issuing the bonds are continuously traded in frictionless markets. The Schwartz model considers commodity price risk, default risk, and interest rate risk and takes the form of a second order partial differential equation in four variables which governs the value of the commodity-linked bond at any point in time. Let \( P \) be the value of the reference commodity bundle, \( V \) the value of the firm issuing the bonds and \( r \) the instantaneously riskless rate of interest and assume that they follow continuous paths described by the following stochastic differential equations:

\[
\frac{dP}{P} = \alpha_p \ dt + \sigma_p \ dz_p \tag{1}
\]

\[
\frac{dV}{V} = (\alpha_v - \frac{D(V,t)}{V}) \ dt + \sigma_v \ dz_v \tag{2}
\]

\(^1\) A commodity-convertible bond will be shown to be equivalent to an appropriately specified commodity-linked bond with "American" options.
\[ dr = \alpha_r(r) \, dt + \sigma_r(r) \, dz_r \]  

where \( D \), the rate of total payouts of all the security holders of the firm (dividends, interest, etc.). \( \sigma_p, \sigma_r \) are constants; \( dz_p, dz_v \) and \( dz_r \) are Gauss-Weiner processes with 

\[
\begin{align*}
    &dz_p \cdot dz_v = \rho_{p} \cdot dt, \quad dz_p \cdot dz_r = \rho_{pr} \cdot dt, \quad dz_v \cdot dz_r = \rho_{vr} \cdot dt 
\end{align*}
\]

The total value of the commodity-linked bond can be expressed as

\[ B = B(P, V, r, T) \]  

where \( T \) is the time to maturity.

If a portfolio is formed by investing

- \( X_1 \) in the underlying commodity, \( P \)
- \( X_2 \) in the firm, \( V \)
- \( X_3 \) in a riskless discount bond, \( C \)
- \( X_4 \) in the commodity-linked bond, \( B \)

\[ X_4 \] in the commodity-linked bond, \( B \)

1/ The commodity-linked bond is a conventional bond with commodity (call) warrants attached to coupon or principal payments.
then the instantaneous total return on this portfolio $dY$ will be

$$dY = X_1 \frac{dP}{P} + X_2 \frac{dV + dt}{V} + X_3 \frac{dG + dB + Cdt}{G}$$

\(G\) is assumed to depend only on \(r\) and \(T\), i.e. \(G(r,T)\); \(c\) is the coupon payment of the commodity bond; by applying Ito's Lemma, we get

$$\frac{dG}{G} = \sigma_G dt + \sigma_G dz_r$$

If we apply Ito's Lemma in (4) and introduce the result in (5) with (1), (2) and (6) and choose \(X_1, X_2, X_3, X_4\) so that the portfolio return becomes riskless, the following partial differential equation governing the value of the commodity-linked bond at every point in time is derived:

\[
\frac{1}{2} \sigma^2 P^2 B_{pp} + \frac{1}{2} \sigma^2 V^2 B_{vv} + \frac{1}{2} \sigma^2 B_{rr} + \\
\sigma_P P V B_{pv} + \sigma_P P B_{pr} + \sigma_V V B_{vr} + \\
r P B + (rV-D) B_v + (\alpha_r - \lambda \sigma_r) B_r - B_T - rB + C = 0
\]

The value of the bonds will be independent of the expected return on the commodity and on the firm; it will only depend on the current values of the reference commodity bundles and the firm \((P,V)\). The promised payment on the bonds at maturity is equivalent to the face value of the bonds \(F\), plus an option to buy the reference commodity bundle at a specified exercise price
The promised payment can be made only if the value of the firm at maturity is greater than that amount.

It is assumed that in case of default, the bondholder takes over the firm. The boundary condition at maturity can be expressed as

\[
B(P,V,r,0) = \min \{V, F + \max (0, P - E)\}
\]  

(8)

As solution of (7) and (8) is very difficult, the following three simplified versions of the model can be obtained:

**Case 1: Uncertain Commodity Price** (No default risk, constant interest rate)

Under the assumptions of no default risk and constant interest rates, the solution of (7) subject to (8) gives

\[
B(P,T) = \frac{C}{r} (1 - e^{-rT}) + Pe^{-rT} + W(P,T)
\]  

(9)

where \(W(P,T)\) is the Black-Scholes solution to the value of a call option with exercise price \(E\). 1/

---

1/ The Black Sholes formula is:

\[
W(P,T) = PN(x) - Er^{-T} N(x - \sigma \sqrt{T}) \text{ where } x \equiv \frac{\log (P/Er^{-T})}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T}.
\]
If C/F is the coupon rate that the issuer must offer to sell the bonds at face value at time of issue T, then (9) can be written as:

$$\frac{C}{F} = r - \frac{r}{(1 - e^{-rT})} \cdot W(P, T)$$

(10)

The issuer has 3 parameters under his control to influence the price of the bond at the time of the issue: the coupon rate (C/F), the exercise price (E), and the amount of the commodity to be included in the reference bundle.

**Case 2: Default Risk** (constant interest rate; only one commodity bond; zero dividends paid)

If S is the total value of equity of firm

i.e. \( V = B + S \)  

(11)

then the boundary condition can be rewritten as

$$S(P, V, 0) = \begin{cases} \max (0, V - F) & \text{for } P \leq E \\ \max (0, V - P + E - F) & \text{for } P > E \end{cases}$$

(12)

Solving (7) subject to (11) and (12), the value of commodity-linked bonds can be derived as

$$B(P, V, T) = V(1 - M_{a1} - M_{b3}) + e^{-rT} F(M_{a2} + M_{b5}) + P M_{b4} - e^{rT} E \cdot M_{b5}$$

(13)
where $M_1$ to $M_5$ are integrals of transformation of $F, V, r, T, E, P$ inter alia.

This bond value includes the default risk in addition to commodity price risk.

**Case 3: Interest rate risk** (no default risk, stochastic interest rates)

If $Q(T)$ replaces the default free discount bond $G(r,T)$ and its growth rate is appropriately replaced in (5), then the derived new (7) function may be solved subject to the boundary.

$$B(P,r,O) = F + \max (0, P - E)$$

If the commodity-linked bond is of the discount type, its value can be expressed as

$$B(P,Q,T) = F - Q + W(P,Q,T) \quad (14)$$

where the value of option $W(P,Q,T)$ can be obtained from Merton (1973).

Several numerical examples by Schwartz (1982) using cases 1 to 3 show interesting properties of commodity bonds. In Case 1 the higher the standard deviation of the commodity price ($\sigma_p$), the higher is the value of the option ($W$) and the lower the required coupon rate ($C/F$). When the value of the reference bundle ($P|F$) becomes zero, the bond becomes riskless and $C/F$ equates to $r$. When $P|F = 1$, i.e., the value of the reference bundle equals the face value of the bond, the equilibrium coupon rate is negative.
In Case 2, the boundary condition indicates that default at maturity depends not only on the value of the firm, but also on the value of the commodity bundle. A higher standard deviation on the return on the commodity \( \sigma_p \) has two opposing effects on bond values: first, it is well known that the value of an option increases with the standard deviation of its underlying security; but second, the probability of default also increases with \( \sigma_p \) and this tends to lower bond values. The first effect dominates the second for low commodity bundle prices, for high firm values and for shorter maturity dates. Default risk has thus a significant impact on bond values and most of this risk comes not from the firm being unable to pay the face value of the commodity bonds, as in the case for regular corporate bonds, but from the firm being unable to pay the value of the option for high commodity prices even under substantial increases in the value of the firm. A higher correlation between the return on the commodity and the return on the firm increases bond values. As the risk of default decreases, the value of the bond approaches the solution for the no default, constant interest rate case.

The analysis involving Case 3 shows that when pricing commodity bonds it is quite safe to use the constant interest rate model as long as the relevant interest rate used is the one to the maturity of the bond.

It is noteworthy that some of the assumptions used to derive the Schwartz models are questionable. The model assumes, for example, that the underlying commodity is perfectly tradeable. The model neglects taxes completely. Also, like most of the option pricing literature, the model assumes constant variances. More complex capital structures and bond characteristics such as call features, sinking funds and convertibility into the reference commodity bundle before maturity if convertible commodity bonds
are considered could be introduced at the cost of having to use complicated numerical procedures to solve the appropriate partial differential operations. The Schwartz model can be also modified to price commodity bonds issued by sovereign states rather than by corporations.
VI. THE FINANCING AND RISK MANAGEMENT NEEDS OF DEVELOPING COUNTRIES 
AND THE ROLE OF VARIOUS RISK-MANAGEMENT INSTRUMENTS

The recent changes in interest rates, exchange rates and commodity prices have adversely affected the revenues and expenses of developing countries, their net worth, their assets and liabilities, and their need and capacity to borrow from domestic and foreign markets. Developing countries tap international financial markets for a number of reasons: (i) to finance projects when domestic funds are not available (in this way growth is accelerated and resources are better used); (ii) to shift consumption inter-temporarily (i.e., developing countries can use financial markets to channel income towards future consumption or to borrow and invest/consume more today in anticipation of higher incomes later); (iii) to take advantage of mispricing of risk/return premia, due to imperfections in capital markets (i.e., when risk premia in the international markets are lower than those of the domestic market, it is possible for foreign investors to invest in domestic projects in pursuit of higher returns not otherwise available in international markets for the same level of risk).

In recent years, financial deregulation, innovation and globalization have made the distribution of funds between sources and uses more efficient. However, despite these important developments in the financial markets, developing countries have not always been provided with, or taken advantage of, the kinds of funding and risk management tools that would be best fitted to their needs. Developing country funding has been mostly on a LIBOR basis, with variable cost, syndicated loans denominated mostly in US dollars. In retrospect, this form of financing has not always been best suited to the characteristics of developing countries.
1. The economies of developing countries are less diversified than those of developed countries. Many are primary commodity-dependent and therefore highly subject to swings in world economic activity. The recent increase in variability of interest rates and exchange rates has exacerbated such swings. The preponderance of floating rate debt has left developing countries highly exposed to the monetary and fiscal policy mix of the United States and other major industrial countries, leading to a highly volatile debt service structures.

2. While the currency composition of developing country debt is primarily dollar-denominated, loans in currencies of other major industrial countries have become more accessible in the last 10 years. It could only be by accident that the currency denomination of developing country debt provided a close hedge to the pattern of their export earnings. In order to reduce the exchange rate risk of borrowing, it is desirable to diversify the currency in which funding takes place to achieve a better match between the currency composition of developing country debt and the currency pattern of export earnings. Such diversification would also reduce the cost and/or the variance of the external debt of developing countries in the long run. This kind of diversification implies the need for making exchange rate swaps (or the use of exchange rate options and futures) accessible to developing countries.

3. The bulk of funding available to developing countries is of a pure debt nature, which places the performance risk exclusively on developing countries. Equity or quasi-equity forms of financing such as commodity-linked bonds create a better balance between the risks and rewards of investments. It is desirable, therefore, to make such instruments more accessible to developing countries.
4. Commercial banks carry most of the credit exposure of developing countries, which increases the possibility of mispricing risk since such pricing is not exposed to the full competitive process of the financial system. In addition, such forms of financing may increase the exposure of a particular bank in a particular country, creating undesirable political problems. For this reason, it is desirable to find orderly means of redistributing some of this credit exposure to other market participants. This could be achieved through the various forms of securitization developed in recent years.

This section describes in more detail the role which the various financial instruments can fill in meeting the risk management needs of developing countries as identified.

1. **Swaps**

Swaps of various kinds could be powerful tools for enabling developing countries to manage their assets and liabilities more effectively. Swaps can be utilized to reduce the volatility of their service payments by achieving a better mix between fixed and floating rate obligations and by diversifying the currency composition of their debt to create a better match between the currency composition of their debt and the structure of their export earnings. Swaps can also be used to change the cash flow pattern of their debt service payments. Finally, they could be used for risk-sharing purposes. This can be achieved through debt/equity or commodity swaps.

(i) **Interest Rate Swaps**

With the bulk of the debt of developing countries on a floating rate basis, use of interest rate swaps would enable developing countries to reduce the volatility of their service payments by converting part of their debt to
fixed interest rates. The interest rate swap market is very liquid (reputed by an annual volume of US$200 billion), thus permitting a significant volume of swaps by developing countries.

While the benefits of increased stability of debt service payments could be significant, there could be two potential drawbacks for developing countries, namely, immediate debt service payments could rise and they could forego benefit from any further declines in interest rates. Developing countries can only be certain LIBOR will be lower than a fixed-rate borrowing for a six-month period. Given the substantial volatility of LIBOR, it could be desirable for developing countries to pay a comparatively small premium for the potential long-term benefits that a fixed-rate form of debt could provide them. Getting into an interest rate swap depends, therefore, on the expectations of financial officers about short and long-term interest rates. For diversification purposes, however, it is important to have a mix of variable and fixed-rate loans. The second concern can be addressed through the use of a mixture of swaps and interest rate options (see below). The latter could guarantee developing countries a ceiling on the cost of their debt. Although options are somewhat more costly, they enable developing countries to benefit from any future downward movements in rates. In short, for many developing countries it could be desirable to convert part of their debt to fixed-rate to diversify their portfolio and thus to protect themselves against any potential rise in rates. Similarly, it could be in the interest of lenders to provide developing countries with a more stable debt service profile. Indeed, it may even be desirable to encourage financial institutions to help developing countries to finance the additional cost of the fixed-rate funds as an inducement to move to a less volatile debt profile.
(ii) Currency Swaps

With the bulk of the debt of developing countries denominated in US dollars, currency swaps can help developing countries restructure the currency composition of their debt to better match the currency pattern of their export earnings. In some developing countries, it may be desirable to reduce the volatility of their debt service ratio by denominating debt in currencies that are linked with their export earnings. Currency swaps enable developing countries to create a better hedge between their debt service payments and export earnings. \(^1\) Independently of that, however, currency swaps can be used to mitigate the cash flow problem of developing countries.

(iii) Debt/Equity Swaps and Commodity Swaps

At present, the bulk of the debt obligations of developing countries take the form of pure debt instruments with the performance risk exclusively carried by the developing countries. Given the particular vulnerability of developing countries to exogenously-determined factors, this form of obligation has proven to be highly burdensome and at times extremely unfair. The promotion of equity or quasi-equity instruments could provide a better balance in the risk/reward aspects of developing country obligations. Increased equity participation in developing countries could take several forms. Part of the commercial banking debt could be transformed directly into

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\(^1\) Other developing countries may be less concerned about creating this long-term hedge and more concerned about the level of their current debt service payments. In this case, developing countries could use the currency swaps to increase the share of low nominal cost currencies, thus reducing their average nominal cost of funding. If the interest rate parity theory were to hold, in the long run such a strategy would be analogous to a shift in their debt from current coupon obligations to obligations with an original issue discount. However, empirical evidence shows that interest rate parity does not always hold.
equity or convertible securities or commodity bonds. New loans could be made under these new forms. It is also possible to create synthetic instruments which represent a continuum between pure debt and pure equity obligations through the use of commodity-linked bonds, commodity-indexed securities, commodity swaps, or long-term commodity options. The successful experimentation of Chile, the Philippines and Mexico with debt/equity swaps reveals another area where modern financial engineering could be instrumental in improving the debt situation of many developing countries. Commodity-indexed swaps can be structured to enable developing countries to service part of their debt through forward sales of primary commodities. In the same way that the currency swap market has led to the development of a forward currency market in long maturities, commodity-indexed swaps could play the same role in commodity markets. Commodity-indexed swaps have the advantage of linking the developing countries' debt service payments directly to their major asset, i.e., commodity resources. They would create a quasi-equity risk/reward relationship which would enable developing countries to diversify away from general obligation financing. By participating in commodity bonds and commodity swaps, developing countries can reduce the volatility of their profits, ceteris paribus. The consequences from a proliferation of these instruments could be extremely important. Evaluation of projects, formulation of budgets and the making of investment decisions could be greatly facilitated. Interest in commodity-linked bond issues has been renewed. Oil and metal swap markets are increasing rapidly; they are expected to pass the billion dollar volume mark this year.
(iv) Other Swap Structures

Swaps into zero coupon involving original issue discounts and amortizing swaps can be structured to meet the specific cash flow needs of developing countries. The credit exposure involved in such approaches tends to be higher than conventional swaps and involves more complex intermediation processes.

2. Options

Given the precarious financial position of many developing countries, it is conceptually very attractive to create various forms of financial insurance schemes that would protect the developing countries against significant adverse movements of interest rates, exchange rates and commodity prices. Alternatively, "insurance" can provide partial protection to the banking system against the prospect of not being repaid. Options may be viewed as a family of "insurance" arrangements. Buying of options provides the purchaser with insurance while the seller of the option is the provider of insurance. They can be a very flexible tool for redistributing risk between borrowers, lenders and potential third parties.

Call options can be used to create interest rate "caps" for developing countries, establishing an upper limit to their interest costs. This reduces the riskiness of their portfolio. Similar to swaps, interest rate caps place a ceiling on borrowing costs. However, unlike swaps, interest rate caps enable developing countries to benefit from declining rates as well; though they do involve an upfront payment which is not required in the case of swaps. The market for caps has grown quickly and annual volumes of US$20 billion or more may be feasible. An alternative could be for developing countries to buy a cap and set a floor on their interest rate (i.e., buy a
spread or "collar"). This would enable them to finance part of the cost of the cap by setting a floor to their interest payments.

Similar caps can be created for exchange rate and commodity price exposure. Using call options, developing countries can lock in a series of future commodity prices. Their earnings performance would then depend only on the volume produced. Stripping commodity options from commodity bonds could increase the depth of the market, making longer-term option trading and hedging possible.

In practice, a whole family of instruments exist which involve the use of call options. By adjusting the strike price of these calls, combining purchases and sales of calls, developing countries can vary the degree of protection they obtain and its cost.

**Put options** can be used to provide current or prospective lenders insurance against the possibility that developing countries are unable to repay their debt. The World Bank, for example, could sell long-dated puts to commercial banks to induce them to lend additional funds to developing countries or to lend for much longer maturities. The put could be structured to have a strike price below par. This would create a disincentive for the commercial bank to put the credit. If it did, however, the put would permit an orderly write-down of developing countries' debt. Again, a whole family of arrangements exist which involve varying striking prices. In addition, various hybrid arrangements can be entered into. For example, a commercial bank could sell an international organization such as the World Bank an "in the money" call, enabling the World Bank to obtain concessionary funding to finance a loan that has been put to the World Bank. Commodity-related puts could also smooth commodity import outlays for developing countries. The
World Bank could intermediate by matching developing countries' demands for commodity puts and calls.

3. Securitization

It may be desirable to promote securitization of developing countries' debt to enable the debt to be more widely distributed in the market. As discussed above, one of the current problems with the state of lending to developing countries is that the source of such funds has been primarily the commercial banking sector. By securitizing such debt and widely distributing the securities two things can be accomplished: first, a better assessment of risk can be ensured and second, a wider distribution of risk can be realized. The benefits of securitization of debt must, however, be carefully evaluated against some of its potential disadvantages, e.g., rescheduling becomes far more difficult and debt write-down may have to be undertaken by commercial banks. Although a general approach to securitization may be undesirable, specific applications, particularly for the more creditworthy countries, may be worthwhile.

4. Commodity-Linked Bonds

The commodity-linked bond is a risk management instrument that offers considerable potential in assisting developing countries to stabilize the prices of their primary commodity exports, in enhancing their credit worthiness, in raising new money, in restructuring their debt, in distributing performance risk more equitably, and in locking in the profitability of projects.

Use of commodity bonds go back over at least a century. However, there has been a recent awakening of interest in commodity-linked bonds. The demand for these issues comes from natural resources funds and from high yield
and other portfolio holders that wish to diversify their risk/return mix as well as actual consumers of the commodities in question. In the last 12 months some 45 commodity bonds (most linked in oil and in precious metals) have been issued in the US and Euro markets. In response to this perceived demand, investment and commercial banks are considering the issue of commodity-linked bonds on behalf of developing countries. But it appears that they will undertake such issues for the less credit-worthy countries only if an international organization such as the World Bank establishes a mechanism to cope with the performance risks involved in developing country participation in this market. The World Bank could, for example, provide commodity loans to developing countries and issue similar commodity bonds in the international markets. The issue of commodity bonds by the World Bank could effectively reduce its cost of financing and thus that of developing countries. Alternatively, the World Bank could strip the pool of commodity loans in two parts (i.e., the conventional loan and the commodity warrant parts) and sell the commodity warrants in the commodity markets. The World Bank could also create mechanisms to allow developing countries to raise funds in the financial markets using commodity bonds.

Commodity bonds can bring multiple advantages to developing countries:

(a) they can provide access to financial markets that would not otherwise have been available. Consequently, commodity bonds may lead to improved creditworthiness of developing countries;

(b) they can provide an opportunity not only for refinancing debt but also for issuing new paper;
(c) they can help overcome cash flow problems of commodity-producing countries by establishing coupon payments lower than those of straight bonds issued at par; they can thus offer more attractive terms to the issuer than might otherwise have been possible. With commodity bonds, producers forego the opportunity gain above the exercise/strike price. In exchange for this foregone opportunity, they receive credit at terms better than they would have otherwise received;

(d) they can be used as risk management tools by commodity producers. Commodity bonds and long-term commodity warrants can smooth commodity proceeds, reduce the variability of profits and net worth and improve investment decision making throughout periods of troughs and peaks. Commodity bonds can link commodity prices (and revenues) to interest or principal payments (expenses). Commodity-producing organizations can thus lock in their profits and partly ensure their ability to service their debt. Although it is not advisable for diversification purposes to link their entire production or output to interest payments, there is an optimal amount that profitable producing companies should aim at locking-in through commodity bond financing. (Clearly this amount is smaller for commodity consumers than for commodity producers). Long-term commodity warrants can lock in commodity prices within a specified range over long periods;

(e) they can be used as risk management tools not only by commodity producers but also by commodity consumers; commodity warrants can be used by commodity consumers to hedge against commodity price increases above a certain striking price level; and
(f) they can improve liquidity in commodity markets by creating a market for longer-term commodity warrants and by increasing the demand for shorter-term commodity options that can be used for book hedging purposes.

It is unlikely that there will be a private sector solution to the perceived problem of the performance risk associated with developing country participation in this market. Therefore, developing country access to this source of finance will remain low unless this obstacle can be overcome. The World Bank is in a unique position to provide a solution, for several reasons: (a) through the salutary sanctions which it exercises as an international institution, the Bank is not exposed to the same degree of moral hazard as private institutions; (b) the Bank is a respected AAA creditor with an extensive knowledge of financial and commodity markets; (c) it is an independent body that represents interests of both developing and developed countries and can therefore act as an "honest broker" between bond issuers, trading houses and buyers of the bonds; and (d) the Bank has the mandate to help developing countries gain access to additional external finance in this way and to provide them with technical assistance to help them deal with their commodity risk management problems.
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